# $AdS_4/CFT_3$ at one loop

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ABSTRACT: I consider semi-classical type IIA strings rotating in the $AdS_4$ part of $AdS_4 \times \mathbb{CP}^3$. The one loop sigma model corrections to this classical solution are used to compute the energy shift, and the result is found to be $E - S = f(\lambda) \ln S$ with $f(\lambda) = \sqrt{2\lambda} - \frac{5 \ln 2}{2\pi} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right)$. Even though the functional forms match, the actual numerical value of this one loop string result differs from the result obtained on the integrable $\mathcal{N} = 6$ Chern-Simons (ABJM) theory side.

# Contents

## 1. Introduction

Recent developments suggest that the worldvolume theory of $N$ membranes[1] on the orbifold $\mathbb{C}^4/\mathbb{Z}_k$ is a certain three-dimensional $\mathcal{N} = 6$ SU($N$) × SU($N$) Chern-Simons-matter theory at level $(k, -k)$. This theory is christened ABJM theory [1], and it arose as a generalization of the ground-breaking work of Bagger, Lambert and Gustavsson. In turn, Bagger-Lambert theory can be reproduced as a special case of ABJM, when the rank of the gauge-group is given by $N = 2$ and the level is small. When $N \gg 1$ and $N^{1/5} \ll k \ll N$, it is possible to argue that ABJM theory is dual to type IIA string theory on $AdS_4 \times \mathbb{CP}^3$ and this is the context we will be interested in. In what follows, we will use $\lambda$ to stand for $\frac{N}{k}$, the t'Hooft coupling of the theory.

A very interesting aspect of these $\mathcal{N} = 6$ Chern-Simons-matter theories is that they seem to be integrable [3, 4]. In particular, a conjecture along the lines of that of Beisert, Eden and Staudacher for $\mathcal{N} = 4$ SYM [5] was made recently for ABJM theory as well [6]. This result is expected to be valid for all values of the coupling. In particular, we can use this to do an in inverse coupling expansion at strong (gauge) coupling. But then, AdS/CFT suggests that we should be able to reproduce the strong coupling expansion by a sigma-model perturbation expansion on the string theory side. In this paper, we will find that even though the form of the expressions match as expected, the precise values of the coefficients do not.

We will consider a class of string states which have large angular momentum $S$ in $AdS_4$ [10 – 12]. The expectation is that these classical solutions correspond to twist-two

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[1]See [2] for recent membrane-related papers.

operators in the dual gauge theory. Their anomalous dimensions are expected to take the form $E - S = f(\lambda) \ln S$ in a wide class of gauge theories [7]. For the case at hand, we can look up the strong coupling prediction for $f(\lambda)$ for ABJM theory from [6] and the result is

$$f_{\text{CS}}(\lambda) = \sqrt{2\lambda} - \frac{3 \ln 2}{2\pi} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right). \tag{1.1}$$

On the other hand, we will see that the result of worldsheet perturbation theory around the classical spinning string solution gives

$$f_{\text{string}}(\lambda) = \sqrt{2\lambda} - \frac{5 \ln 2}{2\pi} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right). \tag{1.2}$$

The second piece comes from the one-loop sigma model corrections. It is possible to argue following [11] that corrections of the form $\sim \ln^2 S$, which could have invalidated this scaling, do not arise, just as they did not in $AdS_5 \times S^5$. So the form of the expression matches the AdS/CFT expectation, but clearly the numerical values differ.

In the next section we introduce the $AdS_4 \times \mathbb{CP}^3$ background and write down the classical rotating string solution with spin in $AdS_4$. In section 3, we compute the one loop correction to the energy. The contribution from the bosonic fluctuations are easily adapted from previous work in the $AdS_5 \times S^5$ context [11], but the fermionic fluctuations require us to inspect the quadratic fermion pieces in the type IIA Green-Schwarz superstring in $AdS_4 \times \mathbb{CP}^3$. Once we fix the masses of the various fields, it is straightforward to compute the energy shift. In the concluding section we make some comments about the result. Some reviews which consider topics of relevance here at an introductory level are, [17].

**Note added.** The author of this note was hesitant to publish these results even after completion because he kept thinking that the disagreement between the gauge theory and the string theory must be due to a computational error. But then, a paper by McLoughlin and Roiban appeared [15] (and also [16] quickly thereafter) in which they get precisely the same result (in the $J = 0$ case) as this paper. That has finally imparted the cowardly author of this paper with the requisite courage to put his results out for public scrutiny.

## 2. IIA on $AdS_4 \times \mathbb{CP}^3$

We will take the metric of $AdS_4 \times \mathbb{CP}^3$ in global coordinates, because the time translation isometry in these coordinates is dual to the scaling dimension of gauge theory operators. So in the string frame we have [8],

$$ds_{\text{IIA}}^2 = R^2(ds_{AdS4}^2 + 4ds_{CP3}^2), \tag{2.1}$$

where

$$ds_{AdS4}^2 = -\cosh^2 \rho \, dt^2 + d\rho^2 + \sinh^2 \rho \, d\Omega_2^2, \tag{2.2}$$

$$ds_{CP3}^2 = d\xi^2 + \cos \xi^2 \sin^2 \xi \left(d\psi + \frac{\cos \theta_1}{2} d\varphi_1 - \frac{\cos \theta_2}{2} d\varphi_2\right)^2$$

$$+ \frac{1}{4} \cos^2 \xi \left(d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2\right) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2). \tag{2.3}$$

Here, we will take the two sphere metric on $AdS_4$ to be of the form $d\Omega_2^2 = d\beta_1^2 + \cos^2 \beta_1 d\phi^2$. The angles on $\mathbb{CP}^3$ run between[2] $0 \leq \xi < \frac{\pi}{2}$, $0 \leq \psi < 4\pi$, $0 \leq \varphi_i \leq 2\pi$ and $0 \leq \theta_i < \pi$. In the full metric $R^2$ is related to the t'Hooft coupling of the Chern-Simons theory though $R^2 = \pi\sqrt{\frac{2N}{k}}$. Note that we have set $\alpha' = 1$, we can reinstate it by replacing $R^2$ with $R^2/\alpha'$ everywhere. The other fields in the background are a constant dilaton, an RR 2-form $F^{(2)} = dA$ threading the $\mathbb{CP}^1$ cycle of the $\mathbb{CP}^3$, and an RR 4-form $F^{(4)}$ on $AdS_4$:

$$F^{(2)} = k\bigg( -\cos\xi\sin\xi d\xi \wedge (2d\psi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2)$$

$$-\frac{1}{2}\cos^2\xi\sin\theta_1 d\theta_1 \wedge d\varphi_1 - \frac{1}{2}\sin^2\xi\sin\theta_2 d\theta_2 \wedge d\varphi_2 \bigg), \tag{2.4}$$

$$e^\Phi = \frac{2R}{k}, \qquad F^{(4)} = \frac{3}{2}kR^2 \, \mathrm{Vol}_{AdS_4} \tag{2.5}$$

Worldsheet fermion mass terms in such backgrounds are generated through the coupling in the Green-Schwarz action to the RR-forms. For this, it is useful to introduce a vielbein basis. In terms of the non-coordinate one-forms, the metric takes the form

$$ds_{\mathrm{IIA}}^2 \equiv \eta_{AB}\theta^A\theta^B, \quad \eta^{AB} = diag\{-,+,\ldots,+\}. \tag{2.6}$$

We will take $A \in \{0,1,2,3\}$ to correspond to AdS and $A \in \{4,5,6,7,8,9\}$ to be tangent to $\mathbb{CP}^3$. After the necessary adjustments in the normalization, the forms take the form (that rhymes!)

$$e^\Phi F^{(2)} = -\frac{1}{R}(\theta^4 \wedge \theta^5 + \theta^6 \wedge \theta^7 + \theta^8 \wedge \theta^9), \quad e^\Phi F^{(4)} = \frac{3}{R}\theta^0 \wedge \ldots \wedge \theta^3. \tag{2.7}$$

The contractions of these forms with Gamma matrices will turn up in the fermion mass matrices, and we write the relevant ones below:

$$\frac{e^\Phi}{4!}F_{ABCD}\Gamma^{ABCD} = \frac{3}{R}\Gamma^{0123} \quad \text{and} \quad \frac{e^\Phi}{2!}F_{AB}\Gamma^{AB} = -\frac{1}{R}(\Gamma^{45} + \Gamma^{67} + \Gamma^{89}). \tag{2.8}$$

## 2.1 Classical spinning string on $AdS_4$

Now we describe the classical spinning string solution in $AdS_4$. The type IIA superstring in the Green-Schwarz formulation can be written as [13]:

$$S = -\frac{R^2}{2\pi}\int d^2\sigma \bigg(\frac{1}{2}G_{mn}\partial_a X^m \partial_b X^n \eta^{ab} - i[\eta^{ab}\delta_{IJ} - \epsilon^{ab}(\sigma_3)_{IJ}]\partial_a X^m \bar{\theta}^I \Gamma_m (\mathcal{D}_b)_{IJ}\theta^J\bigg) \tag{2.9}$$

For now, we will work in the conformal gauge, which means that the Virasoro constraints have to be imposed additionally. The spinors are Majorana-Weyl and they are two in number, so $I, J$ run over $\{1, 2\}$. The fact that the $I, J$ indices are placed up or down is irrelevant. Because type IIA is non-chiral the spinors have opposite chirality. The derivative $\mathcal{D}$ is the one from the Killing spinor equation of IIA supergravity, pulled back

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[2]For discussions on the geometry of projective spaces and their fibered spheres, see e.g., [9].

on to the worldsheet. For the case when only the 2-form and the 4-form are present in the background, it becomes:

$$(\mathcal{D}_a)_{IJ} \equiv \partial_a X^M D_{MIJ} = \partial_a X^M \left( \partial_M + \frac{1}{4} \omega_{AB\,M} \Gamma^{AB} \right) \delta_{IJ} +$$

$$+ \frac{e^\Phi}{4} \partial_a X^M \left[ \frac{1}{2 \cdot 2!} F_{AB} \Gamma^{AB} (i\sigma_2)_{IJ} + \frac{1}{2 \cdot 4!} F_{ABCD} \Gamma^{ABCD} (\sigma_1)_{IJ} \right] \Gamma_M, \quad (2.10)$$

The spinning string solution is simple enough, and essentially identical to the one explored in [10], but we will write down some of the details here for use in the next section. For the metric written down in the last section, the solution takes the form

$$t = \kappa\tau, \ \phi = \omega\tau, \ \rho = \rho(\sigma) = \rho(\sigma + 2\pi), \ \xi = \frac{\pi}{4}, \ (\kappa, \omega \text{ const.}) \quad (2.11)$$

while the rest of the coordinates are set to zero. The equation of motion and the conformal gauge constraint imply

$$\rho'' = (\kappa^2 - \omega^2) \sinh\rho \cosh\rho, \quad (2.12)$$

$$\rho'^2 = \kappa^2 \cosh^2\rho - \omega^2 \sinh^2\rho. \quad (2.13)$$

The periodicity of the $\rho$ coordinate can be imposed by a singly folded string (See [11]). This means that

$$2\pi = \int_0^{2\pi} d\sigma = 4 \int_0^{\rho_0} \frac{d\rho}{\rho'}, \quad (2.14)$$

where $\rho'$ can be determined using the Virasoro constraint above, and $\rho_0$ is determined by $\rho'|_{\rho=\rho_0} = 0$.

Classically the energy and spin of the solution are given by

$$E = \frac{\partial L}{\partial \dot{t}} = \frac{R^2 \kappa}{2\pi} \int_0^{2\pi} d\sigma \cosh^2(\rho) \equiv \sqrt{2\lambda}\, \mathcal{E}, \quad (2.15)$$

$$S = -\frac{\partial L}{\partial \dot{\phi}} = \frac{R^2 \omega}{2\pi} \int_0^{2\pi} d\sigma \sinh^2(\rho) \equiv \sqrt{2\lambda}\, \mathcal{S}. \quad (2.16)$$

where dots stand for derivatives with $\tau$ and $L$ is the Lagrangian: $S \sim \int d\tau L$. This implies that

$$\frac{\mathcal{E}}{\kappa} - \frac{\mathcal{S}}{\omega} = \pi. \quad (2.17)$$

## 2.2 The long string limit

We are interested in the long string limit because that is where we expect to make semi-classical contact with twist-two operators in the gauge theory [10]. This coresponds to $\frac{\omega^2 - \kappa^2}{\kappa^2} \ll 1$, i.e., in situations of interest, we will be able to set $\omega \approx \kappa$. The Virasoro constraints imply that then we may set $\rho' \approx \kappa$ as well. Using the conserved charge forumlas of the previous section and the turning point equation (2.14), it is straightforward to show that

$$E - S = \sqrt{2\lambda} \ln S. \quad (2.18)$$

This is a classical result. Quantum corrections could potentially change this to

$$E - S = f(\lambda) \ln S + g(\lambda) \ln^2 S + \ldots \tag{2.19}$$

One of the claims in the gauge-string matching is that the higher powers of logarithms all vanish, because the classical solution is supposed to be dual to gauge theory operators whose anomalous dimensions are expected to have no higher log corrections. The scaling function $f(\lambda)$ is expected to be computable from a strong coupling expansion on the gauge theory. In the next section, we will compute the one loop sigma model correction to the energy. The form of the corrections ties in with the gauge theory predictions, but the one-loop correction to $f(\lambda)$ (i.e., the next order term after the tree level result $\sqrt{2\lambda}$ found in [1]) does not.

## 3. Quantum corrections in the sigma model

To compute the one-loop shift in energy, we need to compute both the masses of the bosonic fluctuations and those of the fermionic fluctuations.

### 3.1 Bosons

We will compute the bosonic masses not in the conformal gauge, but by using the Nambu-Goto action and imposing the static gauge. The results in this case have already been worked out in [11] for the $AdS_5 \times S^5$ case, and we can easily adapt their results. In static gauge, we can impose

$$\tilde{t} = 0 = \tilde{\rho}, \tag{3.1}$$

where tilded quantities denote fluctuations. Writing out the Nambu-Goto action for the fields (including the fluctuations), expanding the determinant, keeping quadratic pieces, and rescaling the fields so that they have a canonical flat worldsheet kinetic term,[3] one ends up with

$$S_B^{(2)} = -\frac{1}{4\pi} \int d^2\sigma [\partial_a \bar{\phi} \partial^a \bar{\phi} + m_\phi^2 \bar{\phi}^2 + \partial_a \bar{\beta}_1 \partial^a \bar{\beta}_1 + m_\beta^2 \bar{\beta}_1^2 + \partial_a \psi_s \partial^a \psi_s] \tag{3.2}$$

where

$$m_\phi^2 = 2\rho'^2 + \frac{2\kappa^2\omega^2}{\rho'^2}, \quad m_\beta^2 = 2\rho'^2. \tag{3.3}$$

Bars over fields denote that they are fluctuations, but after a rescaling so that the kinetic terms are canonical. Of the eight fields, the six fluctuations along the $\mathbb{CP}^3$ (denoted by $\psi$) are all massless.

In the long string limit, we can treat the masses as roughly constants. At the folding points of the string where $\rho'$ runs to zero, there are some subtleties, but it is possible to use conformal invariance to claim that they do not invalidate the arguments below [11].

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[3]This is allowed because of conformal invariance.

### 3.2 Fermions

The quadratic part of the fermionic Green-Schwarz action contributes to the energy shift:

$$L_F = i[\eta^{ab}\delta_{IJ} - \epsilon^{ab}(\sigma_3)_{IJ}]\,\partial_a X^m \bar{\theta}^I \Gamma_m (\mathcal{D}_b)_{IJ}\theta^J \tag{3.4}$$

The structure of the fermions is again very similar to the one found by Frolov and Tseytlin in [11]. We will set $R = 1$ in the following because in all the quantities that we wish to keep track of, they cancel (See eq. (2.8)). This is because the dialton always multiplies the RR-forms in all couplings, and because only the relative factors between the kinetic and potential pieces of the fermion action are important for our mass calculation.

There are two differences in our case as opposed to that of [11]. One is that the "mass term" takes a slightly more complicated form due to the RR-forms, and the second more important difference is that the fermions are of opposite chirality. The worldsheet gamma matrices can be defined as in $AdS_5 \times S^5$: in particular, the rotation needed to remove the $\sigma$-dependence of the worldsheet Gamma matrices takes exactly the same form. This means that we can use many of the results of [11] essentially directly. A further simplification results because we are working in the long-string limit, where the worldsheet covariant derivatives reduce to ordinary ones. (cf. eqn (5.34) of [11]). When the dust settles, the "kinetic" part can be written as

$$i[\eta^{ab}\delta_{IJ} - \epsilon^{ab}(\sigma_3)_{IJ}](\bar{\Psi}^I \tau_a \partial_b \Psi^J) = \tag{3.5}$$
$$= -i\rho'(\bar{\Psi}^1(\Gamma_0 - \Gamma_1)\partial_0\Psi^1 + \bar{\Psi}^1(\Gamma_0 - \Gamma_1)\partial_1\Psi^1 + \bar{\Psi}^2(\Gamma_0 + \Gamma_1)\partial_0\Psi^2 + \bar{\Psi}^2(\Gamma_0 + \Gamma_1)\partial_0\Psi^2).$$

Here the $\tau_a$ are as defined in [11]: $\tau_a = \rho'(\Gamma_0, \Gamma_1)$. Notice that we can use the conformal invariance to get rid of the overall factors of $\rho'(\approx \kappa$ in the long-string limit). This will result in a scaling of the masses.

Now we turn to the potential part. This involves the coupling of the RR-forms, and using the results of the section 2, this can be written as

$$i[\eta^{ab}\delta_{IJ} - \epsilon^{ab}(\sigma_3)_{IJ}]\frac{1}{8}\bar{\Psi}^I \tau_a \Big[ -(i\sigma_2)_{JK}(\Gamma^{45} + \Gamma^{67} + \Gamma^{89}) + 3(\sigma_1)_{JK}\Gamma^{0123}\Big]\tau_b\Psi^K$$
$$= \rho'^2\frac{1}{4}\Big[ -\bar{\Psi}^1(\Gamma^{45} + \Gamma^{67} + \Gamma^{89})(1 + \Gamma^{01})\Psi^2 + \bar{\Psi}^2(\Gamma^{45} + \Gamma^{67} + \Gamma^{89})(1 - \Gamma^{01})\Psi^1$$
$$-3\bar{\Psi}^1\Gamma^{0123}(1 + \Gamma^{01})\Psi^2 - 3\bar{\Psi}^2\Gamma^{0123}(1 - \Gamma^{01})\Psi^1 \Big]. \tag{3.6}$$

Now, using the fact that the spinors are of opposite chirality, we combine them into one spinor $\Psi = \Psi^1 + \Psi^2$, where $\Gamma\Psi^1 = \Psi^1$ and $\Gamma\Psi^2 = -\Psi^2$, with $\Gamma = \Gamma^{0\ldots9}$. This is a standard trick, see for example [14]: in fact, we could have started with IIA fermion action written in this form using Majorana spinors instead of Majorana-Weyl spinors. The advantage of doing this is that here a natural $\kappa$-symmetry gauge fixing choice becomes obvious,[4] namely, $\Gamma^{01}\Psi = \Psi$. Under all this, the kinetic term simplifies to

$$-i\rho'(\bar{\Psi}(\Gamma^0\partial_0 - \Gamma^1\partial_1)\Psi), \tag{3.7}$$

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[4]Note that the $(\Gamma^0 \pm \Gamma^1)$ factors in the fermion kinetic term can be rewritten as $\Gamma^0(1 \mp \Gamma^{01})$.

and the potential term becomes

$$\frac{i}{4}\rho'^2\bar{\Psi}(-(\Gamma^{45}+\Gamma^{67}+\Gamma^{89})\Gamma+3\Gamma^{0123})\Psi. \tag{3.8}$$

We can scale by $\rho'^{1/2}$ in the kinetic term, calculate the eigenvalues of the mass matrix, and we find that there are two massless fermions and six fermions with masses $\rho' \approx \kappa$.

### 3.3 The energy shift

Now we are in place to put everything together. Again, we can follow the lead of Frolov and Tseytlin for the $AdS_5 \times S^5$ case to compute the correction to the energy. We will look at the masses from the previous sections in the long string limit. Here, $\rho' \approx \kappa \approx \omega \approx \frac{1}{\pi}\ln S \gg 1$. This means that our masses simplify considerably (in fact, we used some of these simplifications already in the computation of the fermionic masses in the last section, as already mentioned). The analog of expression (6.6) in [11] then takes the form

$$\begin{aligned}
\Delta E &= \frac{1}{\kappa}\sum_{n=1}^{\infty}\left[\sqrt{n^2+4\kappa^2}+\sqrt{n^2+2\kappa^2}+4\sqrt{n^2}-6\sqrt{n^2+\kappa^2}\right] + \mathcal{O}(1/\sqrt{\lambda}) \\
&\approx \int_1^{\infty}dx\left[\sqrt{x^2+4\kappa^2}+\sqrt{x^2+2\kappa^2}+4\sqrt{x^2}-6\sqrt{x^2+\kappa^2}\right] \\
&\approx -\frac{5\ln 2}{2\pi}\ln S + \mathcal{O}(1/\sqrt{\lambda})
\end{aligned} \tag{3.9}$$

which is our final result. As stated in the introduction, the numerical factor does not seem to agree with the results obtained on the gauge theory side using its integrability.

It should also be noticed that the argument that the shift in energy does not get corrections of the form $\ln^k S$ for large $\kappa$ goes through exactly as in $AdS_5 \times S^5$ because it is based on UV finiteness of the worldsheet theory and dimensional analysis, and not on the specific details of the theory.

## 4. Comments

The computations in the previous sections give rise to a result that is at least superficially unexpected. So it stands to reason whether the discrepancy can be argued to follow from (somewhat more) general grounds. Here we will give an argument that does not depend crucially on the specific fermion masses we computed.

We start by emphasizing that the bosonic masses can be deduced just from knowing the answer in the more familiar $AdS_5 \times S^5$ case. The spinning string in our case rotates on an $AdS_3$ in $AdS_4$. This is entirely analogous to the case in $AdS_5$. In the static gauge, the "transverse mode" is indeed exactly the same, while there is only one extra massive mode (coming from the other directions of the $AdS_4$). It is easy to see from the structure of the fluctuation action that the masses of these two are exactly the same as they were in $AdS_5$. There are six massless modes coming from the $\mathbb{CP}^3$. So overall we got contributions of the form,

$$\sqrt{x^2+4\kappa^2}+\sqrt{x^2+2\kappa^2}+6\sqrt{x^2}, \tag{4.1}$$

in the one-loop integral. This should be contrasted with the bosonic part of equation (6.6) in [11].

Now, we turn to the fermions. The states of the type IIA string theory should be thought of as arising from an orbifolding of M-theory on $AdS_4 \times S^7$ [1]. So the supergravity spectrum can be obtained by the projection from $AdS_4 \times S^7$ onto $\mathbb{Z}_k$-invariant states (where $\mathbb{Z}_k$ is the ABJM orbifold). The fermions are originally in the $\mathbf{8}_c$ of the R-symmetry group SO(8), and decompose as $\mathbf{6}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_{-2}$ under the SU(4) × U(1) R-symmetry group of the ABJM theory. Since the classical spinning string solution we have considered in this paper does not break the symmetries of $\mathbb{CP}^3$, we expect that the fermionic fluctuations should fall into these reps. In particular, we expect that the eight fermions will split into two groups, each containing equal mass fields: the first group will have six fermions and the other will have two. (The two fermions have to have the same mass because of the U(1) charge exchange symmetry.)

These symmetry considerations, together with the fact that the one loop energy shift must be finite, puts stringent restrictions on the energy shift. Lets parametrize the fermion masses by

$$m_6^2 = \alpha \kappa^2, \quad m_2^2 = \beta \kappa^2. \tag{4.2}$$

where the symbols $m_i$ have an obvious meaning. Then, comparing with the masses of the bosons, for finiteness, we need

$$3\alpha + \beta = 3. \tag{4.3}$$

Now, it can be shown by direct computation of the mass shift integral, that for the leading term in its $1/\kappa$ expansion to match with the gauge theory result, one needs

$$\frac{3}{2}[\alpha \ln \alpha + (1 - \alpha) \ln(3 - 3\alpha)] = \ln 2, \tag{4.4}$$

which is (numerically) solved by $\alpha = 0.167721$. But it is easy to see that $\alpha$ should in fact be a pretty "reasonable"[5] rational number because it comes from the square of the eigenvalues of the RR-coupling gamma matrix combination. But explicit computer-based scans have failed to find a rational number $p/q$ that approximates this value of $\alpha$ for positive integers $p$ and $q$ less than 1000, to within the accuracy of the original numerical solution. This (admittedly somewhat handwaving) argument lends further credence that the negative result that we found for the gauge-string match by explicit computation is correct. Even more to the point, the computation itself is fairly straightforward, so it seems difficult to see where we could have gone wrong.

So it will be very interesting to understand what causes this discrepancy from the Bethe ansatz point of view.

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[5]"Reasonable" in this context means that neither the numerator nor the denominator of the rational number are likely to be too large.

## Acknowledgments

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